

Morphological Complex Networks: Can Individual Morphology Determine the General Connectivity and Dynamics of Networks?

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ABSTRACT: This article discusses how the individual morphological properties of basic objects (e.g. neurons, molecules and aggregates), jointly with their particular spatial distribution, can determine the connectivity and dynamics of systems composed by those objects. This problem is characterized as a particular case of the more general shape and function paradigm, which emphasizes the interplay between shape and function in nature and evolution. Five key issues are addressed: (a) how to measure shapes; (b) how to obtain stochastic models of classes of shapes; (c) how to simulate morphologically realistic systems of multiple objects; (d) how to characterize the connectivity and topology of such systems in terms of complex network concepts and measurements; and (e) how the dynamics of such systems can be ultimately affected, and even determined, by the individual morphological features of the basic objects. Although emphasis is placed on neuromorphic systems, the presented concepts and methods are useful also for several other multiple object systems, such as protein-protein interaction, tissues, aggregates and polymers.

‘...the functional superiority of the human brain is intimately linked up with the prodigious abundance and unaccustomed wealth of forms of the so-called neurons with short axons.’
(Santiago Ramón y Cajal, Recollections of my life)

1 Introduction

Look around and all you see is geometry. Although string theory suggests the universe is 10-dimensional, our perception is constrained to 3 spatial dimensions which, jointly

with time, accounts for the 4 traditional dimensions. By contrast, a non-geographical graph (i.e. a graph where the nodes do not have defined positions) lays in an infinite dimensional space. One of the most important consequences of finite-dimensional spaces is the fact that the adjacencies imposed by such spaces, jointly with the presence of spatial constraints (e.g. impenetrability) and the fact that real objects are finite, tend to severely limit long range connections and interactions. For instance, the only way to eat a pineapple is by cutting the fruit somehow. Even when interactions are allowed to go through space, as is the case of electromagnetic fields and waves, such effects tend to diminish steadily with distance. At the same time as such constraints limit our abilities, they are simultaneously responsible for controlling the rate of dynamical changes in the universe, possibly contributing to the enhancement of complexity.

One of the consequences of the spatial constraints and distance metrics typically found in finite-dimensional spaces is the emergence of shape and morphology (e.g. [1, 2, 3, 4, 5, 6]). For the purposes of this work, a *shape* will be henceforth understood as any spatially limited and connected object [6], while *morphology* will stand for the study of shapes. Systems obtained by combining, through connection or not, objects or shapes will be understood as *morphic multiple object systems* — *MMOS*. Examples of shapes include the geographical space occupied by objects such as molecules, cells, organs, and so on. Combinations of molecules give rise to multiple object systems such as materials, combinations of cells to organs, while combinations of organs and tissues produce living beings. Observe that the definition of shape is relative and somewhat arbitrary. For instance, we may be interested in the own shape of MMOS, such as the shape of the heart or lungs, or in the shapes of each constituent object.

The *shape-function* paradigm, whose origin is lost in the history of science (going back to ancient Greece, especially Pythagoras, at the very least), suggests that there is a close interplay between shape and function in nature. For instance, the shape of proteins, together with field interactions, is essential for defining their docking potential. At the same time, the wing of an airplane or a bird is essential for obtaining proper aerodynamics, while the effectiveness of our lungs is directly related to the intricacy of the bronchial channels, which allows enhanced surface area of contact with air. Although the shape-function paradigm can be considered for the investigation of almost every natural phenomenon, the current article concentrates attention on multiple object systems which rely on the geometry of its basic components in order to obtain proper behavior. An immediate example of such systems is provided by the skeletons of vertebrates, which are intrinsically suited to perform the mechanical activities of such animals. A more sophisticated example, which will be central in our discussion, are the biological neuronal networks such as those found in the mammals brain.

The brain is composed by a myriad of cells — the neurons, whose morphology is highly specialized to make *selective* connections. Interestingly, neurons do not connect indiscriminately between themselves, but form an intricate system of short to long-range connections with specific targets [7]. In spite of continuing efforts, the complete understanding on how neuronal connections are established remains a considerable challenge. Current knowledge indicates that neuronal wiring is mediated by every sort of field interactions, including electrical, neurotrophic factors, gradients of ionic and molecular concentrations, gravity, among other possibilities. To any extent, the neuronal milieu represents a nice example of how prolongations (i.e. dendrites and axons)

are required from neurons in order to overcome the adjacency constraints imposed by the finite 3 spatial dimensions. At the same time, the degree of spatial occupancy and complexity of a dendritic tree is directly related to its potential as a synaptic target for growing axons. The immediate consequence of such effects is the fact that neuronal cells tend to have a most diverse and specialized overall morphology, ranging from simple bipolar cells to highly elaborated Purkinje cells.

The complexity of neuronal shapes is immediately substantiated by the fact that there is no current agreement between scientists regarding the number of morphological classes of neurons, with competing alternatives suggesting from 2 to thousands of types. It is therefore hardly surprising that the diversity of neuronal cells identified by Ramon y Cajal in the beginning of the 20th century would be understood by he, the father of modern neuroscience, as a possible explanation for the complexity of human behavior and intelligence [8]. Since then, several evidences about the relationship between neuronal shape and function have been provided, including the congruence between the electrophysiological (i.e. functional) and morphological classes of retinal ganglion cells [9] as well as the correlation between the structure of the receptive fields of those cells and the respective morphology [10]. However, despite such cumulative evidence about the interplay between neuronal shape and function, most artificial neural networks, such as the perceptron [11] and Hopfield models [12], have not incorporated the neuronal morphology.

The mammals brain thus provides what is possibly the most compelling illustration of a morphic multiple object system whose dynamics is, to a substantial extent, the byproduct of the shapes of the individual elements (e.g. [13]). In other words, the shape of the individual components becomes essential for obtaining patterns of connectivity which are required for producing, through electrophysiological means, function and behavior of great complexity. Although *complexity* remains a somewhat elusive concept [14], the primate brain is characterized by a continuum of complexity flowing from the sophistication of the *shape* of its basic elements (i.e. the neuronal cells) to the complexity of the emerging *dynamics*, passing through the complexity of *connections* between such cells. As illustrated in Figure 1, the geometrical properties of the individual neuronal cells define the connectivity between those cells, which ultimately produces complex dynamics. Interestingly, such effects are bilateral. For instance, the shape of neuronal cells is changed both as a consequence of stimuli presentation, which promotes the appearance of new neuronal processes and synapses, as well as by evolutionary forces acting during long periods of time.

Interesting as it can be, biological neuronal systems are but an example of how the geometrical properties of individual components can affect, or even determine, the emerging connectivity and dynamics of multiple object systems. This article provides an overview of such an important and relatively overlooked paradigm from the following five perspectives: (a) how to measure relevant geometrical properties of shapes; (b) how to obtain stochastic models of classes of shapes; (c) how to simulate systems of multiple morphological objects; (d) how to characterize the connectivity and topology of such systems in terms of complex network concepts and measurements; and (e) how the dynamics of such systems can be ultimately influenced and determined by the individual morphological features of the basic constituent elements. Although emphasis is placed on neuromorphic systems, the problem of protein interaction is also briefly

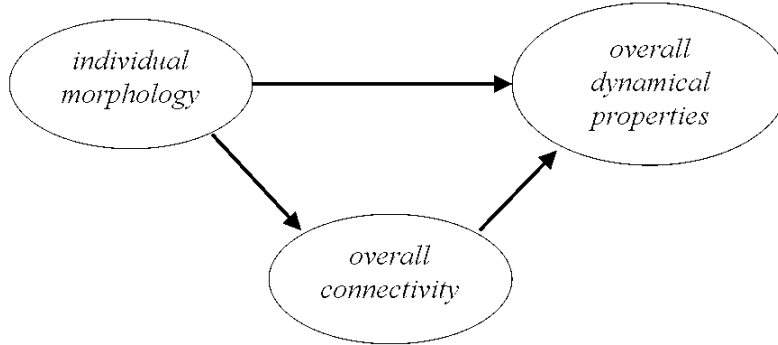


Figure 1: The individual morphological features of basic objects can affect the overall dynamical properties of multiple object systems to which they belong either directly or by determining the overall connectivity between the objects. Very often, as in neuronal systems, the influences are two-sided, in the sense that the overall dynamics may also affect the connectivity and shape of the individual constituent objects.

discussed from the above stated perspectives.

It should be noted that this work is relative to the oral presentation of the author at the COSIN Final Meeting in Salou, Spain, which took place in March 2005. Emphasis is placed on integrating several related previous developments by the author under the shape-function paradigm, so that this text should by no means be treated as a comprehensive survey to related topics. As a consequence, great part of the supplied references are to developments of the author over the last decade, so that the reader is encouraged to refer to the supplied references for a more comprehensive literature.

2 Shape Measurement and Characterization

Given any shape, it is possible to map its morphological properties into a number of measurements, which can be suitably represented as a *feature vector* [6], namely a vector in R^N where each entry corresponds to each measurement of an object under analysis. For instance, given the shape of a neuronal cell, one may be interested to quantify its volume, orientations [15], fractal dimension [16, 17], lacunarity [18, 19, 20], wavelet features [21, 22], curvature and bending energy [23, 16, 17], excluded volume and autocorrelations [24], as well as families of Minkowski shape functionals [25], to name but a few among many other possibilities.

Given such a plurality of alternatives, a fundamental question arises: what is the best set of measurements? The answer to this question should take into account the purpose of the measurements. For instance, in case we are interested in quantifying the overall cell metabolism, a potentially useful measurement would be the total volume of the cell. Two of the most relevant situations in neuromorphology concerns the choice of

sets of measurements capable of: (a) distinguishing between neuronal cell classes and (b) provide a comprehensive representation of the cell geometry, in the sense that the cell shape could be recovered from the measurements, i.e. the set of measurements provides a *complete* representation of the neuronal shape. However, even if the objectives are clearly stated in each of these cases, the optimal set of features is still relative to the specific types of neurons under analysis. For example, in case one wants to separate large from small cells, it may be enough to consider the *diameter* of the cells, namely the largest distance between any pair of points obtained from the cell. Yet, there are some properties of measurements which are generally sought in both situations (a) and (b) mentioned above, especially invariance to translation and rotation. Such properties, which are found in several features such as area, perimeter, curvature, among others, are important because often the shapes in MMOS can appear freely translated and rotated and we do not want the measurement to take this into account. Another relevant issue to be kept in mind is the fact that the set of features should be minimal.

As the choice of features for shape representation will be covered in more detail in the next section, we present in the following some general reasonings on the selection of features for the purpose of shape discrimination or geometrical pattern recognition (see, for instance, [26, 6]). Here the task consists in, given one or more shapes, to organize them into classes such that an object inside each class tends to be more similar to other objects in that same class than to objects from the other classes. Observe that this general problem of pattern recognition has some interesting similarities with the problem of community detection in complex networks. Note that we may or may not know the number of classes, and that we may have or not have examples of objects from each class. In case we do have examples of objects from all the classes, or information about the properties of those objects, we say we have a *supervised* pattern recognition problem; otherwise, it becomes a *unsupervised* pattern recognition case [26, 6]. Needless to say, the first type of problem is generally easier to be solved than the second type.

Unfortunately, the choice of features for pattern recognition is still largely subjective in the sense that there are no definite rules which can be applied for that end. The problem is complicated by the fact that there is an infinite number of features allied to the fact that a large number of measurements do not necessarily lead to better discrimination (e.g. [6]). In practice, one has to rely on previous experience about the problem under analysis, as well as with the properties of the several available features. The remainder of this section presents some brief remarks about some representative shape measurements and their specific interpretation and characteristics.

Hierarchical angles and lengths: Provided the shape can be organized along hierarchical levels, namely as a branching pattern, an interesting set of measurements can be obtained by taking into account the angles and arclengths of the constituent segments (e.g. [27, 28]). The hierarchical decomposition, which is more viable when applied to branching patterns, can be obtained by applying skeletonization approaches as discussed in [29, 30, 31, 32, 33, 34, 35] or by curvature-based methods as described in [36]. In the case of neuronal cells or shapes which have a body, it is important to detect such a body [35] in order to have the correct identification of the beginning of each process (i.e. a dendrite or axon).

Curvature: Quantifying the rate of change of the tangent angle of a given curve, the curvature provides invariance to translation and rotation and is complete (i.e. in-

versible) up to such transformations. The identification of high curvature points provide valuable resources for identifying extremities along shapes [15, 36]. An effective approach to numerical curvature estimation has been described [37, 23, 36, 16, 6, 38] which estimates the shape derivatives required for curvature calculation by using the derivative property of the Fourier transform, combined with Gaussian regularization (i.e. low-pass filtering). The histogram of curvatures of a shape, as well as its respective statistical moments, may also provide valuable information for shape characterization and discrimination.

Bending energy: Given a differentiable two-dimensional shape, its bending energy can be shown to be proportional to the sum of the squared curvature values. Therefore, the more intricated the shape is, the higher its bending energy will result. The bending energy has been used with encouraging success for classification of neuronal cells [23].

Fractal dimension: The several available definitions of fractal dimension provide an interesting alternative for quantifying the ‘complexity’, spatial distribution or spatial coverage of a given shape (see, for instance, [6, 17]).

Lacunarity: Introduced in order to complement the fractal dimension [18, 19, 20], the lacunarity quantifies the degree of translational invariance of a pattern. Typically, the higher the lacunarity, the less translational invariant the shape is. The traditional way to calculate the lacunarity involves sliding a window along the whole image while estimating the mass comprised by the window. Several sizes of windows are usually considered, yielding the lacunarity as function of the windows size. A recent study [39] has shown that translational and rotational invariance can be achieved by using circular windows centered only at the object points, not through the whole workspace.

Minkowski functionals: As implied in the name, shape functionals are maps from a given shape to scalar values. Minkowski functionals are a special class of functionals which are additive, motion invariant and continuous [40, 41, 42]. In the plane, such functionals comprise the perimeter, area and Euler number, corresponding to the number of holes in the object under analysis. Minkowski functionals have been successfully applied to the characterization of neuronal shape [25, 43].

Critical percolation densities: Given a shape and an empty workspace, the progressive superposition of this shape at uniformly distributed random positions along the workspace eventually lead to percolation, which can be identified by looking for a cluster of shapes extending from side to side of the workspace [44]. The average density of shapes observed when percolation occurs, called henceforth the critical percolation density, has been suggested [44] as the most direct measurement of the potential of the given shape for establishing connections, leading to promising discriminative potential. Interesting previous investigations of percolation in neuronal systems, where the neuronal shape was simplified as circles, have been reported in [45]. The critical percolation density can also be estimated for ballistic deposition, as investigated in [46]. In the case the shapes change with time, as is the case with growing neurons, it is possible to define a critical percolation density which is a direct consequence of the growth dynamics and spatial distribution of the cells [47]. In the case of a collection of not necessarily connected and static objects, it is still possible to force percolation through some imposed growth dynamics, such as parallel dilation [48, 49, 50].

Relative shape measurements: While all the above measurements are specific only to the shape of interest, it is also important to consider geometrical features that

also take into account environmental constraints such as the presence of other objects, as well as fields and other types of interactions. One example of such measurements would be to consider the angle difference between the tangent field orientation along the object, which is a measurement intrinsically related to the shape, and the electric field orientation at those same points induced by the presence of surrounding objects. Low averages of such ratios would therefore indicate that the shape in question is highly affected by the external objects. Such measurements are particularly important for studying and modeling MMOS whose basic objects are known to be affected by environmental influences, as is the case with neuronal systems.

3 Stochastic Shape Modeling and Synthesis

In this section we discuss the problem of how to produce, by mathematic-computational means, artificial instances of shapes of a given category. For instance, one may be interested in obtaining a collection of neuronal cells which are statistically equivalent to those of a certain biological class (e.g. alpha cat retinal ganglion cells). By *statistical equivalence* we mean that the synthesized cells will produce the same statistical distribution of values of geometrical measurements. Observe that, given the almost unlimited statistical variability of the properties of natural shapes, even within the same category of shapes, the problem of shape equivalence in the sense of exactly reproducing a given shape is of minor interest. On the contrary, it becomes more important to devise synthesis methodologies capable of incorporating the stochastic variability found in nature.

Two important situations arise: (i) the original shapes evolve without influences from the environment and (ii) the shapes are affected by the environment. The problem of stochastic shape synthesis in these two situations are discussed in the following subsections.

3.1 Absence of environmental influences

In this case, the shapes of interest are the result of a growing dynamics which takes into account only internal constraints imposed by the physics/biology of the object. While there are no natural examples of such growth dynamics, this approximation provides a first, simple approach to modeling and simulating shapes. In the following we illustrate this kind of strategy with respect to the simulation of neuronal outgrowth [27, 28]. Other approaches to neuromorphic modeling include those adopting differential equations to model cytoskeleton outgrowth [51, 52, 53, 54], methods based on Hillman’s [55] set of quantitative anatomical correlations [56, 57], activity and competition during process outgrowth [45, 58, 59, 60] as well as hidden Markov models [61].

The overall idea of the conditional probability model [27, 28] involves obtaining representative measurements from the real biological cells, such as the angles and lengths along the branching hierarchies, represented as conditional densities, and then Monte Carlo sampling such distributions in order to produce synthetic neuronal cells. Figure 2 illustrates three stages along the development of simulated alpha and beta

cells. In both cases, percolation is observed between the growth stages illustrated in the figure. Although this example considers identical cells, which reduces the variance of the percolation critical density, simulations assuming different neuronal cells have also been investigated.

Given a collection of adult neuronal cells, each of them can be imaged in 2D or 3D by using several types of microscopy, including optical transmission and confocal. Although the following discussion is limited to 2D neuronal shapes (several neuronal types, such as retinal ganglion cells, are mostly planar), the extension to 3D is immediate. The image of the neuronal cell can then be processed in order to obtain the points belonging to the cell and the points belonging to the background. The contour of such cells is particularly important and can be obtained by using edge detection and contour following algorithms (e.g. [6]). The particularly important points of the neuronal ramification, namely the extremity and branching points, can be obtained by applying a curvature-based procedure [62, 36] or skeletonization algorithms [31, 6]. The angles and arclengths along each segment along the hierarchies of the ramifications can then be obtained by using simple analytical geometry concepts and methods, so that conditional probability densities of the angles and lengths can be obtained. As discussed in the previous section, such measurements provide, by themselves, a naturally suitable characterization of the neuronal geometry. Note that the number of hierarchies considered in the conditional densities is limited by the amount of neuronal samples, in the sense that the consideration of several hierarchies implies higher dimensionality of joint densities which require larger numbers of cells in order to obtain statistically representative results (i.e. a statistical sampling problem). Finally, the conditional densities can be Monte Carlo sampled in order to produce a virtually infinite number of synthetic cells whose geometrical features will lead to statistical densities equivalent to those which were sampled.

3.2 Presence of environmental influences

The more realistic situation of shape development under environmental influences requires a modeling approach capable of integrating *both* internal and external influences, because the former are always present in physical and biological systems. Often, external influences manifest themselves in terms of *fields* or *waves*. For instance, the growth cones (namely the tips of growing axons) are known to follow gradient fields derived from neurotrophic growth factors, ionic concentrations, electric, and even gravitational fields. Therefore, it is reasonable to incorporate the effect of such fields in neuronal outgrowth by adding to the instant growth velocity (implied by internal influences) a vector component parallel to the external field (e.g. [63, 54, 53]). It has been verified [64] that the incorporation of a vector component pointing away from the cell soma also accounts for biological realism in neuronal outgrowth.

Because the neuronal activity during neuronal development can also have effects in shaping the neuronal morphology, it is interesting to consider such dynamics in simulations. We can use the recently introduced Sznajd complex networks [65], whose topology is determined by correlations between the neuronal connectivity/activity, in order to implement Hebbian dynamics, where the most active connections are reinforced. Another effect to be considered is the fact that waves of ionic concentration

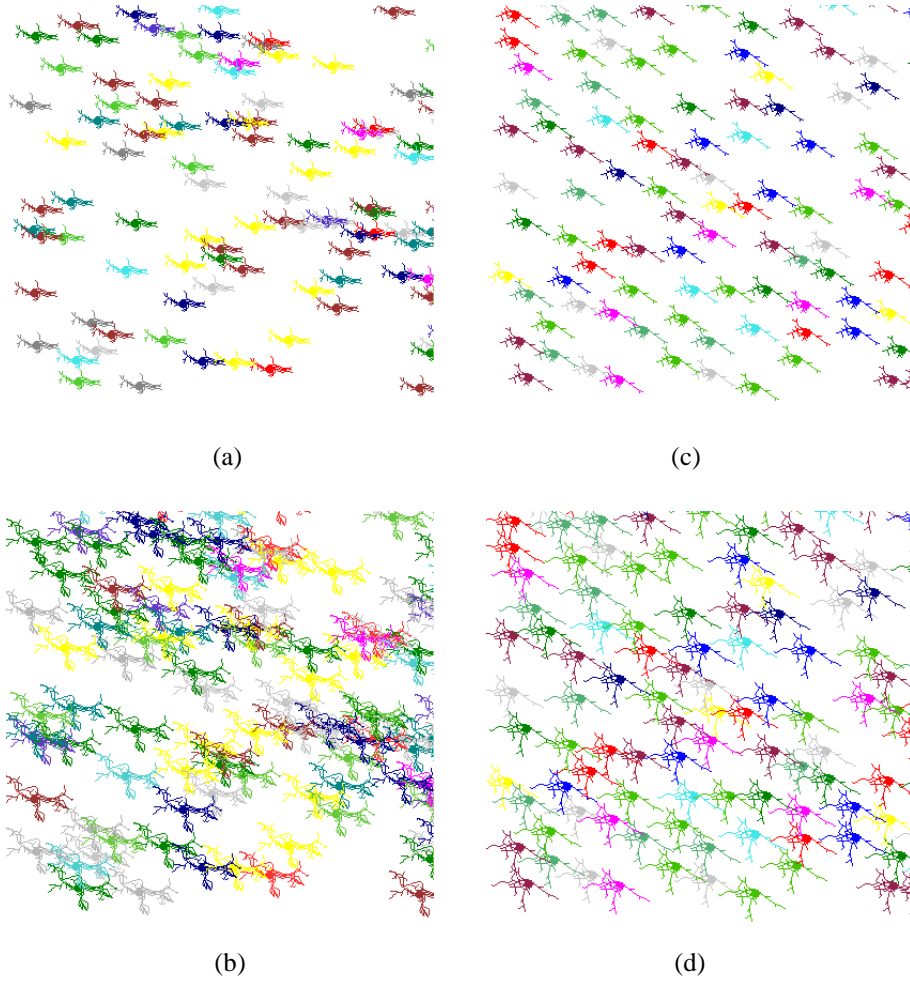


Figure 2: Three stages along neuronal growth simulations for alpha (a-b) and beta (c-d) cells. In this example, all cells are identical.

have been verified to induce neuronal spikes [66]. Therefore, it would be interesting to include such waves in neuronal outgrowth simulations.

4 Synthesis of Morphic Multiple Objects Systems and their Representation and Characterization in Terms of Complex Networks

Provided we have implemented the means to grow individual shapes, they can be combined in order to produce morphic multiple object systems. The most natural means to do that involves starting the growth at N seeds distributed spatially according to some desired statistical density. In the case of neurons, it is often reasonable to assume a uniform distribution of cells (see [67] for a discussion on the measurement of spatial dispersion in neuronal mosaics), which are then grown by using a combination of the techniques discussed in the previous section. Every time an axon gets with a pre-specified distance from a dendrite, it may be connected to that dendrite with some probability which may be a function of the distance as well as other factors (e.g. diameter of the axon and type of cells, among many other possibilities) [27, 28].

Fortunately, MMOSs can be effectively represented in terms of graphs/networks and their respective adjacency or weight matrices [27, 68, 69, 47]. As such systems grow, it is possible to keep track of every new connection (a synapse in the case of neuronal systems), which can be updated into the adjacency or weight matrix of the graph used to represent the MMOS. In the case of neuronal networks, such a representation is almost complete, except for the fact that it does not take into account fields and waves emanating from the environment [66]. However, such effects can be easily incorporated by using geographical networks for representing the system, therefore providing information about the spatial position of the nodes and allowing the determination of the fields at those points.

One of the most interesting perspective in applying complex network measurements for characterization and analysis of growing MMOS regards obtaining each measurement as a function of time or development epochs. For instance, following traditional complex network measurements such as the node degree and clustering coefficient in terms of time can provide valuable information about the local connectivity of the objects as the system evolves. Virtually every complex network measurement will have some additional aspect to say about the evolving connectivity. More global information about the system connectivity can be gathered by using the recently introduced hierarchical versions of the node degree and clustering coefficient, as well as several new hierarchical measurements [70, 71, 72] and wiring lengths between neuronal modules [73]. The application of community finding algorithms can also provide particularly interesting information with implication to the functional dynamics of the MMOS under analysis, with important implications to the evolution of synchronization and attractors formation. Another promising perspective is the quantification of the cycles of several lengths in the growing networks, which will also have important implications for the system dynamics.

The most direct quantification of the evolution of the overall connectivity of the

system is provided by the critical percolation density. We have applied static and dynamics, natural and forced percolations [49] in order to investigate the influence of the individual shape of neurons to the overall connectivity [44, 47]. In investigations considering alpha and beta cat ganglion cells with normalized sizes, we observed that more complex cells tend to percolate sooner. However, the critical percolation density or time (in the case of growing systems with constant density) has been verified to be a function of several other parameters, such as the neuronal cell elongation, the excluded volume, as well as the straightness of the neuronal processes. Further investigations are required in order to establish a clear relationship between the individual object properties, as well as their growth dynamics, with the final overall MMOS connectivity.

5 Consequences on the Overall System Dynamics

Because the individual morphological properties of the objects have been found to determine, jointly with their spatial distribution, both the local and global connectivity of MMOSs, such geometrical features will also be inexorably related to the emerging dynamics of the system. After investigating the effect of using scale free connectivity in Hopfield networks [74, 75], we have been investigating how the individual neuronal shape can affect neuronal dynamics [68]. We have quantified the memory potential of the neuronal systems in terms of the *overlap*, which is related to the number of correct recovered bits, between the originally trained pattern and the pattern recovered (starting from a perturbed version of the original pattern) after P patterns had been trained into the neuronal system. Considering small neurons with the same area, we have found [69] that such a memory potential is highly dependent on the individual neuronal shape, with shapes exhibiting ramified and broader distribution of mass (i.e. neuron-like) tending to perform substantially better than simple shapes such as bars and crosses. The distribution of complex eigenvalue of the adjacency matrices obtained with neuron-like cells have resulted less degenerated, which is known to be associated with better recall capabilities [76].

6 Conclusions and Future Works

The present text provided an integrated review and discussion on the problem of relating shape and function through connectivity in morphic multiple object systems (MMOS). The main paradigm here is the fact that the geometric features at the individual level can play a decisive role in defining the overall connectivity of the system, and consequently its dynamical operation. While the quantification of geometrical features of the individual objects can be achieved by considering a variety of measurements, we have argued that the resulting connectivity of the respective MMOS can be effectively quantified and modeled in terms of complex network concepts and measurements.

Although the shape/function paradigm has been clearly identified and investigated in preliminary works, a series of important problems remain open to research. Of particular interest would be to identify how each of the main shape measurements are correlated to the overall connectivity, as quantified by complex network measurements.

It would also be interesting to develop further analytical models such as that presented in [69] so that the several topological properties of the MMOS can be foreseen from the geometrical features of the constituent objects. One particularly interesting approach would be to try to relate the fractal dimension of the individual neuronal cells with the fractal dimension of the overall topology of complex networks [77], and then with scale-free laws and other properties of the emerging connectivity and activity dynamics of the respective networks. Other promising perspectives include the analysis of recall in neuronal systems other than Hopfield, as well as the quantification of synchronization between cells as the MMOS grows. The most realistic situation where the connectivity dynamics is related to the network activity dynamics along time and space remains one of the most interesting and challenging situation for simulations and investigations. We are working on such issues and would highly appreciate to consider collaborations.

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